

Series of O.D.E.

حل المعادلة التفاضلية باستخدام المتسلسلة.

$$1 \cdot y'' + P(x)y' + Q(x)y = F(x)$$

① O.P

$$P(0) \neq \infty$$

$$Q(0) \neq \infty$$

$$y = \sum_{n=0}^{\infty} a_n x^n$$

Solve 2

ex $y'' + y = 0$, $x_0 = 0$

$$P(x) = 0 \neq \infty , Q(x) = 1 \neq \infty \} \text{ O.P }$$

$$y = \sum_{n=0}^{\infty} a_n x^n$$

$$y' = \sum_{n=0}^{\infty} n a_n x^{n-1}$$

$$y'' = \sum_{n=0}^{\infty} n(n-1) a_n x^{n-2}$$

$$\sum_{n=0}^{\infty} n(n-1) a_n x^{n-2} + \sum_{n=0}^{\infty} a_n x^n = 0$$

Co. of $x^0 \rightarrow 2a_2 + a_0 = 0$

$$a_2 = \frac{-a_0}{2} \rightarrow ①$$

Co. of $x^1 \rightarrow 3 \times 2 \times a_3 + a_1 = 0$

$$a_3 = \frac{-a_1}{6} \rightarrow ②$$

Co. of x^n

$$(n+2)(n+1)a_{n+2} + a_n = 0$$

$$a_{n+2} = \frac{-a_n}{(n+2)(n+1)}$$

at $n=2$

$$a_4 = \frac{-a_2}{4 \times 3} = \frac{+a_0}{24}$$

الحل

$$y = a_0 + a_1 x - \frac{a_0}{2} x^2 - \frac{a_1}{6} x^3 + \frac{a_0}{24} x^4 + \dots$$

b

المسألة

$$(1) \quad t = x - x_0 \longrightarrow x_0 \neq 0$$

$$(ex) \quad y'' + xy' + y = 0 \quad , \quad x_0 = 1$$

$$t = x - 1$$

$$y'' + (t-1)y' + y = 0 \quad , \quad t_0 = 0$$

$$y = \sum_{t=0}^{\infty} a_n t^n$$

(2) شرط في المسألة ١.

نحل معادى وفي الآخر الأمر نكتب الثوابت a_0, a_1

(3) كثيرة الحدود ١.

نساوي معاملات الطرق اليسرى والحد العام.

(4) دوال مثلثية أو لوفاريتمية أو أسية ١.

نفكها.

$$(ex) \quad y'' + y' - xy = 0 \quad , \quad y(0) = 2 \quad , \quad y'(0) = 1$$

$$P(x) = 1 \quad Q(x) = -x \quad \} \quad \underline{0 \cdot P}$$

$$y = \sum_{n=0}^{\infty} a_n x^n$$

$$y' = \sum_{n=0}^{\infty} n a_n x^{n-1}$$

$$y'' = \sum_{n=0}^{\infty} n(n-1) a_n x^{n-2}$$

$$\sum_{n=0}^{\infty} n(n-1) a_n x^{n-2} + \sum_{n=0}^{\infty} n a_n x^{n-1} - \sum_{n=0}^{\infty} a_n x^{n+1} = 0$$

Co. of x^0

$$2a_2 + 1a_1 = 0$$

$$a_2 = \frac{-a_1}{2} \quad \text{--- ①}$$

Co. of x^1

$$6a_3 + 2a_2 - a_0 = 0$$

$$a_3 = \frac{-2a_2 + a_0}{6}$$

بالعوض من ①

$$a_3 = \frac{a_1 + a_0}{6}$$

Co. of x^n

$$(n+2)(n+1)a_{n+2} + (n+1)a_{n+1} - a_{n-1} = 0$$

$$a_{n+2} = \frac{-(n+1)a_{n+1} + a_{n-1}}{(n+2)(n+1)}$$

at $n=2$

$$a_4 = \frac{-3a_3 + a_1}{4 \times 3} = \frac{-\frac{a_0 + a_1}{2} + a_1}{12}$$

$$a_4 = \frac{-a_0}{24} + \frac{a_1}{24} \quad \times$$

∴ the solution

$$y = a_0 + a_1 x - \frac{a_1}{2} x^2 - \frac{a_0 a_1}{2} x^3 + \left(\frac{-a_0}{24} + \frac{a_1}{24} \right) x^4 + \dots$$

$$\text{at } x=0 \rightarrow y=2$$

$$\text{at } x=0 \rightarrow y'=1$$

$$\therefore 2 = a_0$$

$$y' = a_1 - a_1 x + \dots$$

ثم نعوض a_1, a_0 في المعادلة الرئيسية

d)

ex $(x^2 + 4)y'' + xy = x + 2$

$$P(x) = 0 \quad \therefore \quad Q(x) = \frac{x}{x^2 + 4} = 0 \neq \infty \quad \} \quad \underline{0 \cdot P}$$

$$y = \sum_{n=0}^{\infty} a_n x^n$$

$$y'' = \sum n(n-1) a_n x^{n-2}$$

$$\sum n(n-1) a_n x^n + 4 \sum n(n-1) a_n x^{n-2} + \sum a_n x^{n+1} = x + 2$$

Co. of x^0 $\Rightarrow (4 \times 2) a_2 = 2$

$$\boxed{a_2 = \frac{1}{4}} \rightarrow \text{II}$$

Co. of x^1

$$24a_3 + a_0 = 1$$

$$\boxed{a_3 = \frac{1 - a_0}{24}}$$

Co. of x^n

$$n(n-1) a_n + 4(n+2)(n+1) \underbrace{a_{n+2}}_{\text{أكبر ما}} + a_{n-1} = 0$$

$$a_{n+2} = \frac{-n(n-1) a_n - a_{n-1}}{4(n+2)(n+1)}$$

at $n=2$ $a_4 = \frac{-2a_2 - a_1}{4 \times 4 \times 3} = \frac{-a_1 - 1/2}{48}$

$$\therefore y = a_0 + a_1 x + \frac{1}{4} x^2 + \frac{1 - a_0}{24} x^3 + \frac{-a_1 - 1/2}{48} x^4 + \dots$$

Ex

$$xy'' + \sin x y = 0$$

$$P(x) = 0$$

$$Q(x) = \frac{\sin x}{x} = \frac{0}{0} =$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

O.P

$$xy'' + \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots \right) y = 0$$

$$y = \sum a_n x^n$$

$$y'' = \sum n(n-1) a_n x^{n-2}$$

$$\sum n(n-1) a_n x^{n-1} + \sum a_n x^{n+1} - \frac{1}{3!} \sum a_n x^{n+3} + \frac{1}{5!} \sum a_n x^{n+5} + \dots = 0$$

$$\text{Co. of } x^0 \rightarrow$$

لا يوجد

$$\text{Co. of } x^1 \rightarrow 2a_2 + a_0 = 0$$

$$a_2 = \frac{-a_0}{2}$$

$$\text{Co. of } x^n \rightarrow (n+1)n a_{n+1} + a_{n-1} - \frac{1}{3!} a_{n-3} + \frac{1}{5!} a_{n-5} - \dots = 0$$

$$\therefore a_{n+1} = \frac{-a_{n-1} + \frac{1}{3!} a_{n-3} - \frac{1}{5!} a_{n-5} + \dots}{n(n+1)}$$

$$\text{at } n=2 \quad a_3 = \frac{-a_1}{6}$$

$$\therefore y = a_0 + a_1 x + \frac{-a_0}{2} x^2 - \frac{a_1}{6} x^3 + \dots$$